Teaching materials

Guide notes 1. Position analysis of a four-bar mechanism

MISCE project

Mechatronics for Improving and Standardizing Competences in Engineering

Competence: Mechanical Engineering

Workgroup: Universidad de Castilla-La Mancha

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This document corresponds to the first lecture for the competence ‘Mechanical Engineering’ using the ‘Four-bar mechanism platform’

Version: 1.0

Date: October 5th, 2023

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1 Objective

The main objective of this lesson is to introduce students to planar kinematics through the analysis of a four-bar mechanism.

The session explores how geometry and assembly configurations affect motion, including the application of Grashof's law. Students will identify and solve position-related kinematic problems using numerical, analytical, and experimental methods.

2 Context

The mechanical system employed is a four-bar mechanism whose movement is provided by a DC-motor. The input signal to this motor is a pulse width modulated signal (PWM) introduced and control through the App designed in MATLAB (see Lesson 0. Introduction to the experimental platform).

This kind of experiment allows to determine the location and orientation of all links based on a given input angle. This lecture helps students to understand the geometric relationships within a closed-loop system and settles the foundation to analyse velocity and acceleration. In general, students gain general insight into the behaviour, configuration and practical operation of mechanical systems.

3 Position analysis of a four-bar mechanism

A four-bar mechanism is a kinematic chain composed of four links, each connected by two kinematic pairs, forming a closed-loop. It is also important to specify the assembly configuration of the mechanism, since more than one configuration may be possible, (see Lesson 0. Introduction to the experimental platform).

The schematic representation of a four-bar mechanism is shown in Figure 1. The double circle symbolizes a rotational pair between two links, where one of them is the fixed link.

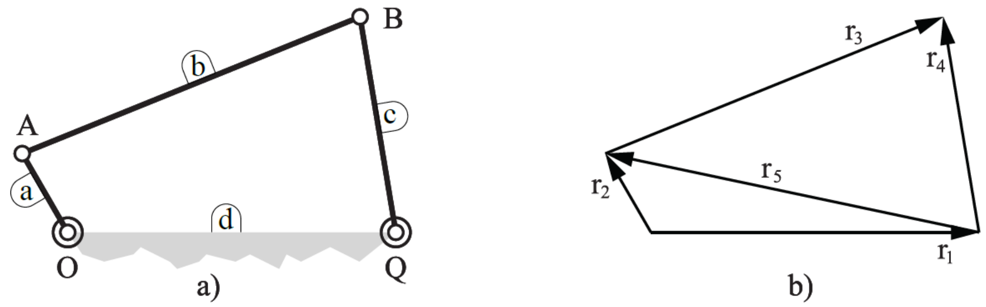

Figure 1. Schematic representation and nomenclature of a four-bar mechanism

The position analysis involves determining the positions and orientations of all links and points of interest in the mechanism, given the geometry of the links and the position of one of them.

To solve the kinematics of the four-bar mechanism, it is necessary to establish the vector loop that models the mechanism. This loop is shown in Figure 1(b). The equation is:

$$\boldsymbol{r_2} + \boldsymbol{r_3} = \boldsymbol{r_1} + \boldsymbol{r_4}$$

Using equation $\boldsymbol{r} = Re(\boldsymbol{r}) + i \cdot Im(\boldsymbol{r})$, explained in the previous document Lesson 0. Introduction to the experimental platform, this can be split into real and imaginary parts, yielding the two scalar equations needed to solve for the two unknowns:

$$\rho_2 \cos(\theta_2) + \rho_3 \cos(\theta_3) = \rho_1 \cos(\theta_1) + \rho_4 \cos(\theta_4)$$
$$\rho_2 \sin(\theta_2) + \rho_3 \sin(\theta_3) = \rho_1 \sin(\theta_1) + \rho_4 \sin(\theta_4)$$

As can be seen, solving for the angles θ_3 and θ_4 leads to a nonlinear system, though this is not a significant obstacle for modern computational tools.

This calculation should be performed both numerically—by solving the equations—and experimentally, using the inclination sensors available in modern smartphones, as well as analytically using trigonometry. To solve the kinematics analytically, an auxiliary vector $\boldsymbol{r_5}$ must first be calculated.

4 Methodology

4.1 Example: Solving the position problem

As an example, the mechanism is solved for the configuration used in this guide, where:

- $\rho_1 = 140$ mm
- $\rho_2 = 50$ mm
- $\rho_3 = 160$ mm
- $\rho_4 = 100$ mm

As an initial condition, an angle of $45°$ is imposed. This can be set using a square ruler or a smartphone. Since $\theta_1 = 0$ and $\theta_2 = \pi/4 = 45°$, solving equations (2) yields:

$$\theta_3 = 0.34 \text{ rad} = 19.47°$$
$$\theta_4 = 1.09 \text{ rad} = 62.48°$$

The MATLAB code for solving the problem is the following:

```
function four_bar_mechanism_position_solver()
% Parámetros del mecanismo
rho1 = 140;
rho2 = 50;
rho3 = 160;
rho4 = 100;
theta2 = pi/4; % Ángulo de entrada (45 grados)
% Suposición inicial para theta3 y theta4 (en radianes)
initial_guess = [pi/4,pi/2]; % [theta3_guess,theta4_guess]
% Resolver el sistema de ecuaciones
options = optimoptions('fsolve','Display','iter');
solution = fsolve(@(x)
mechanism_equations(x,rho1,rho2,rho3,rho4,theta2),initial_guess,options);
% Extraer soluciones
theta3 = solution(1);
theta4 = solution(2);
% Convertir a grados para visualización
theta3_deg = rad2deg(theta3);
theta4_deg = rad2deg(theta4);
% Mostrar resultados
fprintf('Solucion encontrada:\n');
fprintf('theta3 = %.2f rad (%.2f grados)\n', theta3, theta3_deg);
fprintf('theta4 = %.2f rad (%.2f grados)\n', theta4, theta4_deg);
% Verificar cierre del lazo vectorial
error = mechanism_equations(solution,rho1,rho2,rho3,rho4,theta2);
fprintf('Error en el cierre del lazo: [%.2e,%.2e]\n', error(1), error(2));
end
```

```
function F = mechanism_equations(x,rho1,rho2,rho3,rho4,theta2)
theta3 = x(1);
theta4 = x(2);
% Ecuaciones de cierre de lazo vectorial con theta 1 = 0
F = [
rho2*cos(theta2)+rho3*cos(theta3)-rho1*cos(theta1)-rho4*cos(theta4);
rho2*sin(theta2)+rho3*sin(theta3)-rho1*sin(theta1)-rho4*sin(theta4)
];
end
```

This code resulted in:

```
four_bar_mechanism_position_solver()

Iteration   Func-count    ||f(x)||^2       Norm of      First-order    Trust-region
                                              Step       optimality          radius
    0           3           2423.64                        4.53e+03               1
    1           6            124.73       0.713049         1.54e+03               1
    2           9          0.188927        0.09525             45.2            1.78
    3          12       8.04642e-07     0.00625982            0.128            1.78
    4          15       2.41115e-18    6.43286e-06         1.98e-07            1.78

Equation solved
fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
Solucion encontrada:
theta3 = 0.34 rad (19.47 grados)
theta4 = 1.09 rad (62.48 grados)
Error en el cierre del lazo: [4.67e-10,1.48e-09]
```

In the code example provided, the initial guess values for the angles are specified. For the configuration used in this manual, if the initial guess is changed to:

```
initial_guess = [pi/4,pi/2]; % [theta3_guess,theta4_guess]
```

Then the result becomes:

$$\theta_{3=} - 0.99 \text{ rad} = -56.81^\circ$$
$$\theta_4 = -1.74 \text{ rad} = -99.82^\circ$$

This is because in this case, the solution corresponds to the alternative assembly configuration of the four-bar mechanism. The four-bar mechanism admits two possible assembly modes. The student can verify that changing the initial guess may lead to a result that does not match the experimental resolution.

However, students will not be required to verify this second solution experimentally, as doing so would require disassembling and reassembling the mechanism on the small test bench.

4.2 Student task

The student should calculate the angles θ_3 and θ_4 for their chosen four-bar mechanism configuration. These calculations must be done numerically, by solving the equations, experimentally, using the sensors available in the experimental platform together with the MATLAB app, as well as analytically through trigonometry. Additionally, the students can verify the results using the inclination sensors of their smartphones.

The chosen mechanism configuration, i.e. the chosen link lengths, must satisfied Grashof's law and be performed for the following three given positions of link 2: $\theta_2 = 135^\circ$, $\theta_2 = 225^\circ$ and $\theta_2 = 315^\circ$.

Finally, the student must verify that the obtained results match the experimental data recorded from the platform.